The geographic space covered with GeoGebra geometry

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This work has been presented at the GeoGebra International Conference, Budapest, Hungary, 23rd-25th of January 2014

Abstract: *The geographic space offers a great deal of mathematical interpretation, considering this space as one immersed in the Euclidian E3 space or in the E2 Cartesian plane. The support for this attitude could be the GeoGebra-software. The software has made some important steps in order to cover the need of modeling the mathematical space, thus one could accept that any transition could be made towards Geography in order to offer a wider perspective upon the Earth`s spectacle. Even if this geographical environment is represented by maps or stereoscopic images, the connection between Geographical studies, Mathematics and GeoGebra-software in the sense of investigation, becomes more and more accepted today. This study is a window that develops a method of investigation and opens some trends in didactics, focused to GeoGebra.*
Keywords: Geographical space, Mathematical model, GeoGebra-software

1. Introduction: Geographic objects can have many proprieties but from the geographer`s point of view, one of these proprieties plays a special role. This propriety is the location of the object, [Ren65]. Locations are usually defined in a two-dimensional coordinate system, but sometimes this identification is not enough. Nowadays, only by accessing the Google Maps site helps everybody resolve the point with a simple "click" on the map presented on the computer screen. In order to discover the length of a route between two points on a map, especially for a new route, the GeoGebra software will allow us import the 2D map from Google Earth (in jpg format) and go further with the algorithm presented by the authors in 2010. For this, we must find with the help of GeoGebra successive points, defining segments or curves which connect the start point and the end

point correctly. Even if we shall try to propose any new route, and not to cross implicitly defined roads, one could say that any GeoGebra project can be accepted and maybe extended to any developed one. For this reason a procedure that will connect the map scale with the GeoGebra graph must be developed. With the same algorithm, one could investigate distances, perimeters or areas on the map, and other characteristics in the 2D space [SA10], and add scaling maps or some didactics for put-output data on a map, [Ver12].

2. **The geographic space covered with GeoGebra geometry**

Both a mathematician and a geographer must be good at visualizing space. Almost all problems in the nature have a geometric interpretation, and being able to visualize and manipulate geometric images in your head can vastly improves your ability to solve mathematical problems or to improve models. The Earth could be approximated to a sphere; the plane could be understood as a 2-dimensional structure, and other things in nature can be interpreted by geometric structures, [Dav11].

In this sense we shall present some interesting 2-D structures on maps which will show us some geometrical interpretations.

The first project comes to analyze the interesting structure of the architectural plan of Braila County in Romania. Braila is situated in the South East Region of the European Union and it is located in the terminal of the Danube river basin. The river will be in contact with Danube Delta from Braila at only 80 km before flowing into the Black Sea. This area is known as Romanian Danube Shipping Sector. Framed in terms of the mathematical modeling, the area of study it is located at the latitude of 45.28 and longitude of 27.97.

The old structure of the street location it is the same today, the same as 650 years ago, even if new streets appeared and the town it is in a permanent extension. The principal characteristic of all the streets it is that it starts from Danube and end to the Danube if we refer to the most important boulevards, and in the same time, other streets are placed on this map as imaginary rays which start from a fixed point, the center of an imaginary circles. This situation made us import the Google imagine on the GeoGebra screen and to construct many circles, placing consecutive points on the boulevards, at least three, and get consecutive circles defined by three points. With this occasion an interesting picture has been obtained and more of this, constructing the centers of these circles an interesting superposition was obtained, (fig.1).

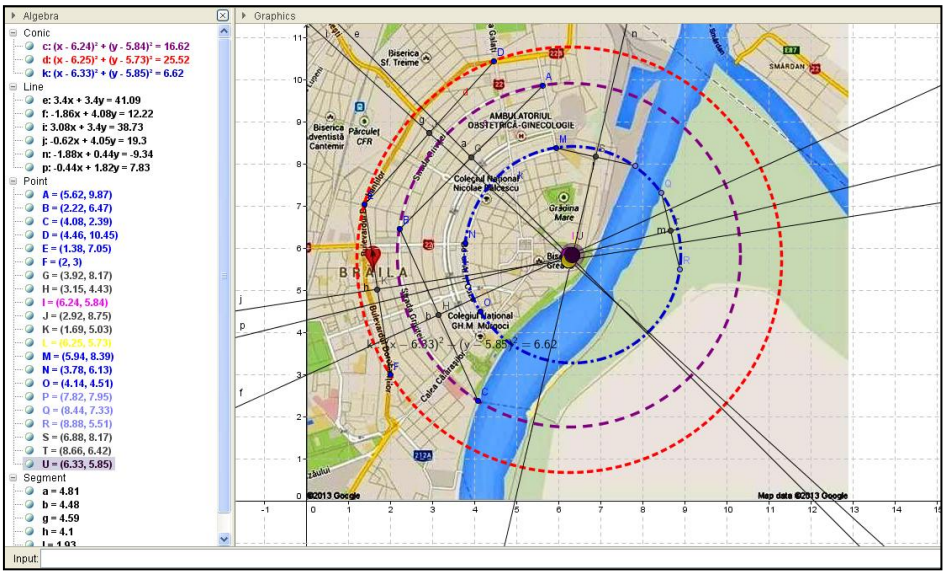


Figure 1. The construction of some circles with the same center over the map of Braila City

In many river cities - Odessa, Bordeaux, Tours, Köln - the architectural plan evolves very similar, from a rectangle to a semicircle bounded by the river. In other cities, situated at the confluence of two rivers - Koblenz, Lyon - the expansions remain very distinct from the original core, [Ben03]. The map of our location, analyzed with GeoGebra for Braila city brings us the idea that the initial rectangle tends to a single point. GeoGebra was applied in the same algorithm for the fourth locations in figures 3-4-5.

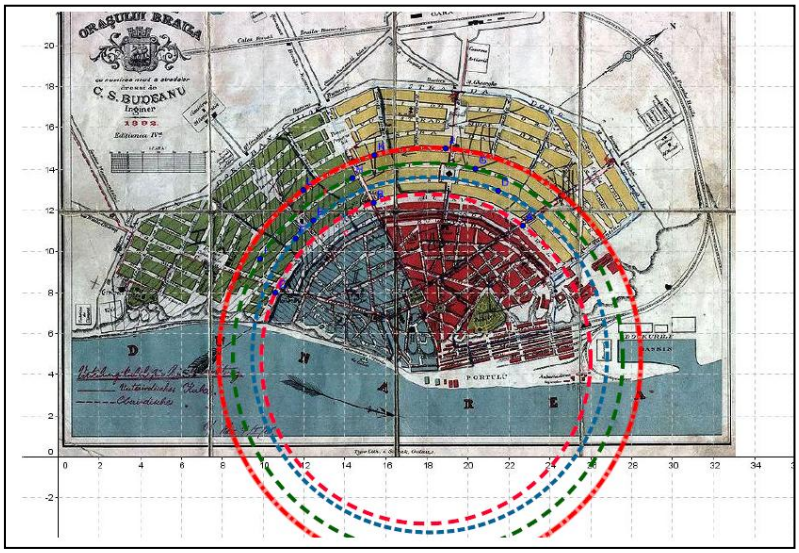
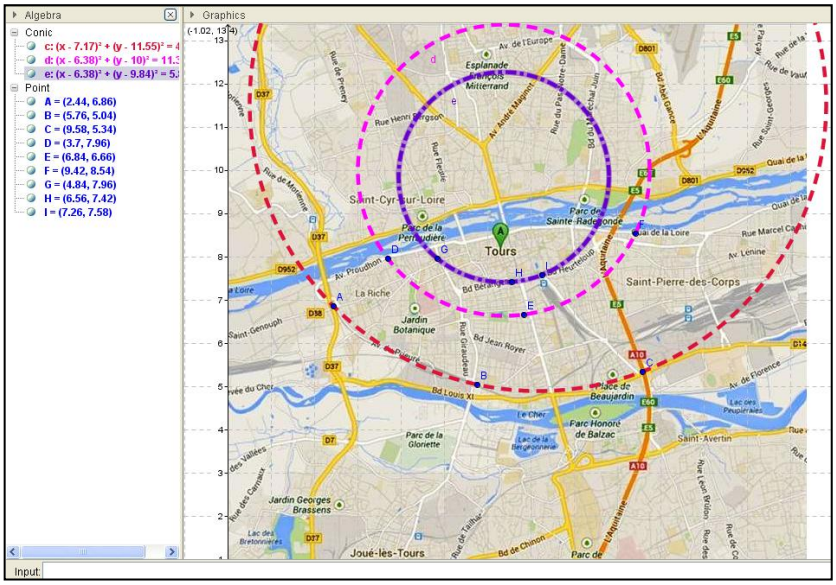


Figure 2. The old map of Braila City year 1892



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c: (x - 7.17)² + (y - 11.55)² = 4
d: (x - 6.38)² + (y - 10)² = 11.3
e: (x - 6.38)² + (y - 9.84)² = 5.
A = (2.44, 6.86)
B = (5.76, 5.04)
C = (9.58, 5.34)
D = (3.7, 7.96)
E = (6.84, 6.66)
F = (9.42, 8.54)
G = (4.84, 7.96)
H = (6.56, 7.42)
I = (7.26, 7.58)
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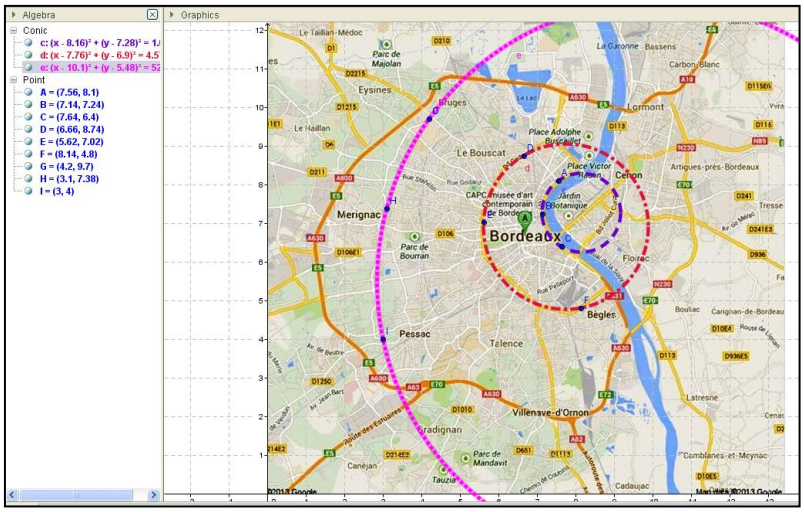
Figure 3. The map of city of Tours in France



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c: (x - 8.16)² + (y - 7.28)² = 1.
d: (x - 7.76)² + (y - 6.9)² = 4.5
e: (x - 10.1)² + (y - 5.48)² = 52
A = (7.56, 8.1)
B = (7.14, 7.24)
C = (7.64, 6.4)
D = (6.66, 8.74)
E = (5.62, 7.02)
F = (8.14, 4.8)
G = (4.2, 9.7)
H = (3.1, 7.38)
I = (3, 4)
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Figure 4. The map of Bordeaux City in France

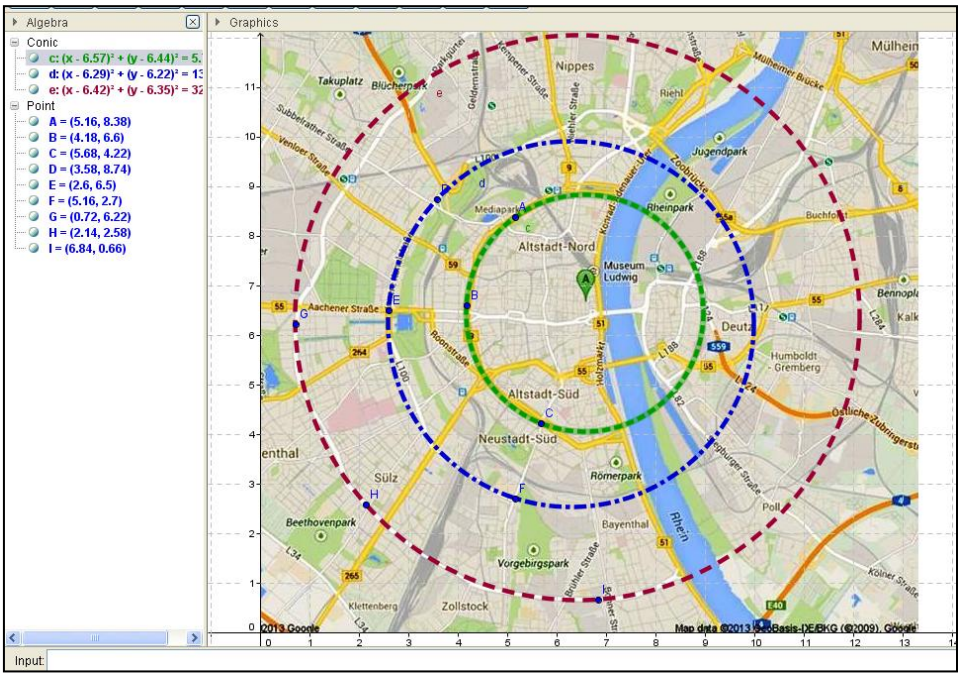

Figure 5. The map of the City of Köln in Germany

3. Didactics trends, focused to GeoGebra

The spectacle of these constructions invites us to open some questions. Some of them could remain opened but, in the same time, there are a lot of new directions that develop new investigations about the visual perception of some 2D maps, about the connections which could be made by the analyzers.

In [Dav11], the author Tom Davis covers a geographical investigation with some strictly geographical questions, inviting the reader to "feel free to add their own questions". Of course all the questions are just suggestions, representing topics of most interest to a geographer which looks for a mathematical interpretation of any kind. After a list of questions referring to latitude, longitude, distances on a map, etc., we shall add some questions referring to our observation and describe the GeoGebra project.

These questions are:

1. How many cities around the world could be covered with geometric structure as circles with the same center, regular polygons with the same center (fig.6), circles with collinear centers (fig.7), or even other more complicated structures?

2. Could this problem be connected with any geographical structure, like for example the position of the city near a river or other corps of water or maybe something about mountains from a proximity, climatically investigated environment or, in a single word, the environment.

3. Which one of the exposed characteristics implies more than other a structure as above? The social structure, the climatic structure or other economical structures could be involved in this analysis.
4. Can we define some algebraically proposed categories as functions with the definition domain as a set of cities around the world and with values in the pre-defined geometrical structures?
5. The list of questions seems to be more complicated and for this reason we shall return to the connection between observation of a 2D space, digital maps and the GeoGebra software connected in the described algorithm of investigation.

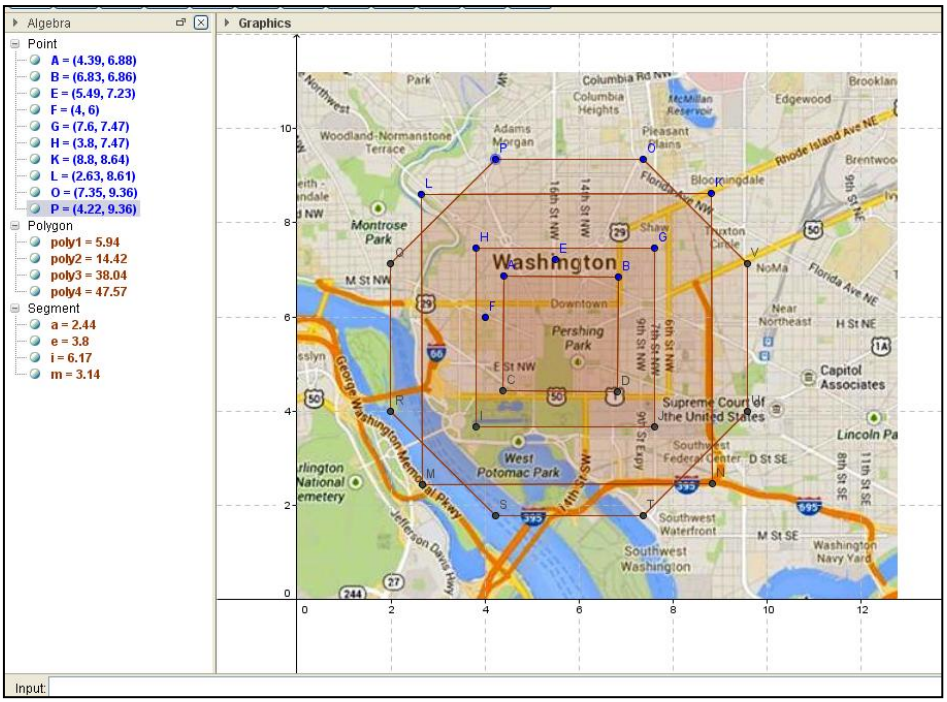

Figure 6. The construction of some regular polygons over the map of Washington DC in USA

If we shall refer to using GeoGebra for our analysis and the facilities which were observed, one could agree that our point of view it is only a start for some interesting exploration.

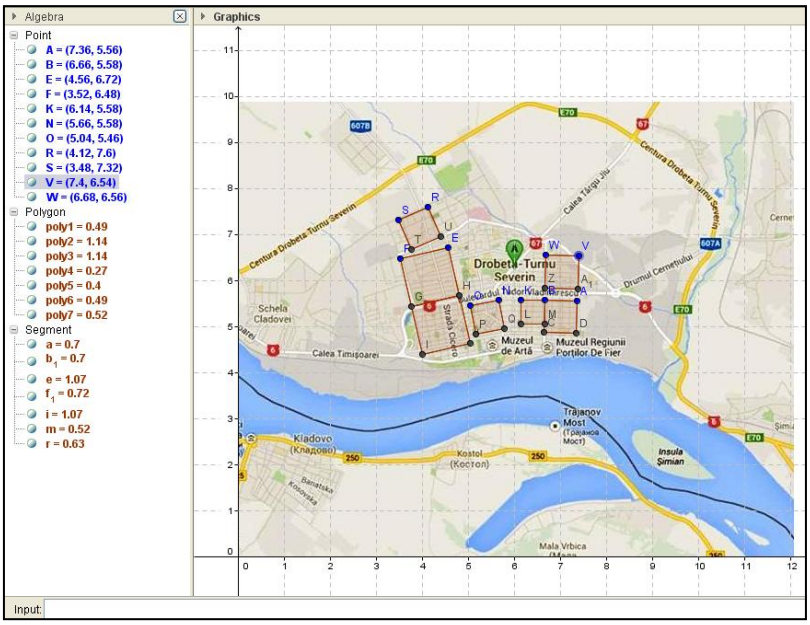

Figure 7. The construction of some regular polygons over the map of Drobeta-Turnu Severin City of Romania,
Similar with Washington DC

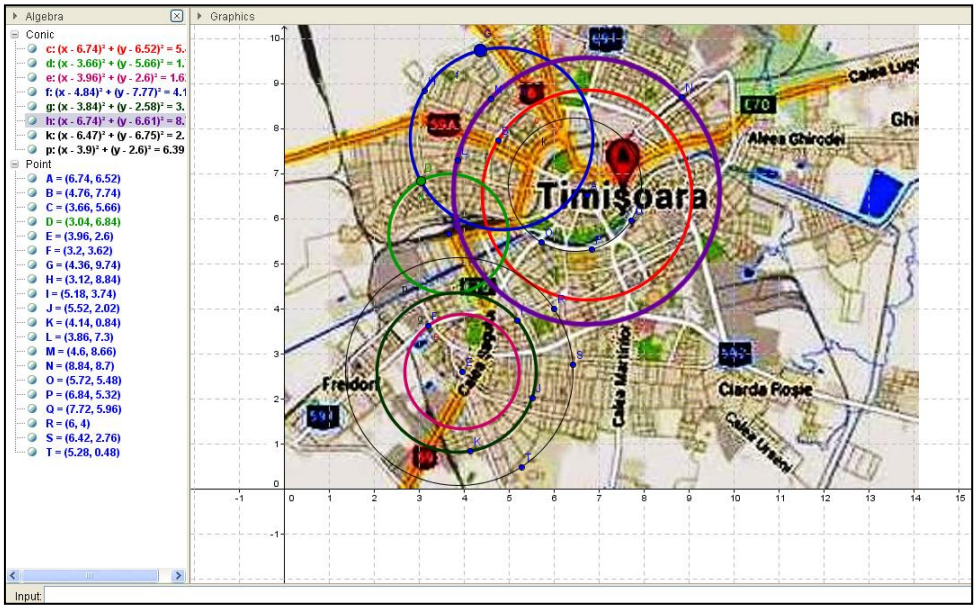

Figure 8. The construction of some circles over the map of Timişoara City of Romania, the location of the first local GeoGebra Institute in Romania, connected with Braila City via GeoGebra.

Conclusions

In our opinion, the study for using the above described algorithm with GeoGebra, will give us the opportunity to open the use of GeoGebra in many others domains, i.e.: Pure Mathematics, Algebra, Analytic Geometry or Mathematical Analysis. For the moment, environmental studies could be considered as an important domain of investigation in order to find out many other proprieties of environmental structures. We have placed here in our discussion the using of GeoGebra and this has given us some new ideas for future environmental studies.

References:

[Ren99] Rene W.O., *"Geometric Algorithms for Geographic Information Studies"*, Ph.D. thesis, Utrecht University, ISBN: 90-393-2049-7, 1999.

[SA10] Soare I., Antohe C., *"Modeling Geographical Studies with GeoGebra-Software",* Annals of Computer Science, Series 8-th, Tome 1-st, Fasc.-1, Pp. 173-180, 2010.

[Ver12] Vera Herceg-Mandic & All, *"The Teaching of Geography Using Dynamic Geometry Software",* Faculty of Science, University of Novi-Sad, Serbia, ISBN: 978-86-7031- 200-5, Pp.11-15, 2012.

[Dav11] Davis T., *"Geometry and Geography",* www.geometer.org 2011.

[Ben03] Benevolo L., *"The town in the European history"*, translation by Lascu M., Polirom Publishing House, ISBN: 973-681-217-1, Bucharest, 2003.