How to Teach Complex Numbers Applying the GeoGebra Software

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Abstract: The paper deals with the application and possibilities of the GeoGebra software for teaching complex numbers and it demonstrates its efficacy in the high school and college programmes. For the software Armenian localization was added. It offers, using the resources of the GeoGebra software when teaching this theme, to include the main theorem of algebra and some of its corollaries into the teaching process and, besides the traditional examples and homework, offers to create some interactive models and tools, which can essentially improve the students' motivation for perceiving and learning the theme.

Keywords: Complex numbers, fundamental theorem of algebra**,** GeoGebra, geometry, mathematics, interactive model.

I have been acquainted with dynamic mathematics software since long ago, firstly coming across them in their Russian variant, the Geometer's Sketchpad. Then I have used the software ArchimedesGeo and Geonext. This software have not much principal differences from the first one, but I was charmed with their new possibilities and advantages they offered for teaching geometry and mathematics and, being not able to stand the temptation, I began to apply them at my classes, simultaneously advertising them among my colleagues. Then I published a number of papers about their possibilities and ways of application, but my greatest wish was to translate them into Armenian. I considered it very important for I wanted to make those wonderful tools accessible to my home citizens, but all the software I knew at the time were not free, and my desire could not be carried out for a long time.

Once I came across the GeoGebra software, which had more functional possibilities and was free. This was the very thing I had wanted, for the rest was depended on my own self. Now it is for two years there has been the GeoGebra software in Armenian. Besides translating the software, I published a book describing GeoGebra's main possibilities, tools, commands and peculiarities [1], in which I presented a number of examples of applications through many practical exercises for the class-work. A CD, containing both the illustration of all examples

in the book and the practical exercises of GeoGebra file possibilities, is attached to the book.

The CD also contains a great many models.

The next step of spreading the software in Armenia was retraining of 10 teachers, subsided by "World Vision Armenia" (see Pic. 1). Then a number of meeting with teachers of the Republic of Armenia followed, which had been arranged by the publisher of my book (see Pic 2, 3). About 1300 teachers took part in the meetings and, believe me, all the teachers were fascinated by the software and they considered its application for teaching mathematics and geometry very important and useful. The only problem the teachers were concerned was the insufficient quantity of devices at Armenian schools, because many schools in Armenia have no sufficient numbers of computers etc.

Pic. 1

Pic. 2

Pic. 3

The softwares of dynamic mathematics, like many other ones, present a certain environment through their tools and commands, and the ways and versions of using this environment generally depend on the abilities of the user. The GeoGebra can be applied when teaching many various themes of mathematics and geometry [2]. My own experience shows that it is useful when teaching physics, too. In particular, I can mention the laboratory for studying the theory of photo-effect [3], which has successfully been applying in the schools.

Now, as I mentioned above, I want to present some ideas of mine about the possibilities and efficiency of GeoGebra when teaching complex numbers.

To match the teaching standards in Armenia with those in Europe the Armenian schools passed to 12 year teaching duration. This transition gave forth certain changes in school mathematics programme in Armenia, in particular, in the classes having science bias the theme, complex numbers, was added [4].

This part of mathematics naturally is rather hard for the school students and this is natural because the theory of complex numbers is 'more abstract' even for those few talented students, especially, if one takes into account the fact that the physical notions must be measurable and, consequently, they must be real, not imaginary or complex.

Besides, in contrast to the real numbers (integers, rational or irrational numbers) which come across in the whole course of mathematics and geometry, after studying the theory of complex numbers the Armenian student never sees them again for the next two final years. Thus, the knowledge about them the student gets is gradually forgotten during the next two years. This is the reason that the subject theme is somewhat isolated and the student has a feeling of aimlessness and unreasonableness when studying these numbers, which results in the lack of sufficient interest and desire for studying them, putting the teacher before a very hard teaching problem, if not an impossible one, because it is much more difficult to teach something when the student has not any motivation.

Now let us see what possibilities there are to put right this situation and to make the teaching and learning of the theme easier.

I believe that the teaching and learning process will essentially gain if:

- the complex numbers ‘become visible,’ that is, if they are displayed in a manner that the operations with them become ‘visibly illustrated;’ moreover, if they become easier and easily changeable
- it is possible, in the frame of the theme, to make judgments about the main theorem of algebra and its consequences, presenting many different examples
- besides the traditional examples and homework in the textbook many interesting and impressive exercises for doing by the computer are offered.

Obviously, it is impossible to carry out the above-said tasks ‘with the chalk on the blackboard’, and the softwares of dynamic mathematics put enormously wider possibilities at one’s disposal, through which one could carry out the above-mentioned three tasks, and GeoGebra is the most convenient free software for all the three tasks, because, in contrast to other softwares of dynamic mathematics, this software possesses the tools and commands for presenting the complex numbers, for finding their sums, differences, products and quotients and for calculating the complex roots of polynomials with real coefficients. The fact that the software is free is an important advantage, and it is the only software of dynamic mathematics that has an Armenian version.

Now let us see, for instance, how the addition of complex numbers is carried out in the GeoGebra environment. Choosing the tool, “Complex Number”, press at some two points on the complex coordinate plane. Then, two complex numbers, z_1 and z_2, are created and the plane becomes a complex one. To get the sum of these numbers write z_1+z_2 in the line, “Input” and press “Enter,” resulting the sum number, z_3, on the complex plane (see Fig. 1).

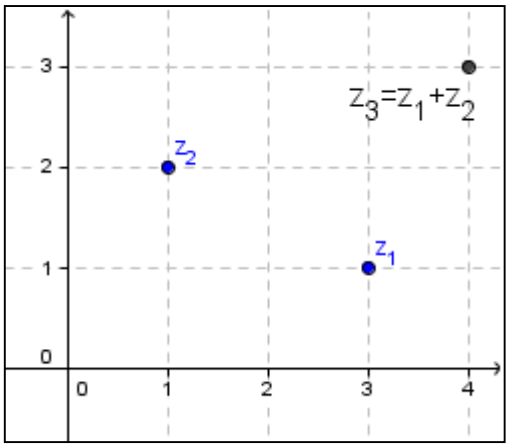

Fig. 1

If one holds the number, z_1 or z_2, by the mouse and changes it, z_3 gets immediately changed, too. Such an ‘addition’ process becomes both interesting and

impressive for the student, promoting rather good perceiving of the geometric illustration of complex numbers.

The figures, 2-4, are fragments of an interactive model illustrating the n-th roots of the complex number, z_0.

Though the software has 70 tools and many commands, nevertheless, it has no tool or command for doing this. Fortunately, there are possibilities of creating such new tools, and one of these possibilities was used to create a tool for finding and illustrating the n-th roots of the complex number and forming an interactive model for them.

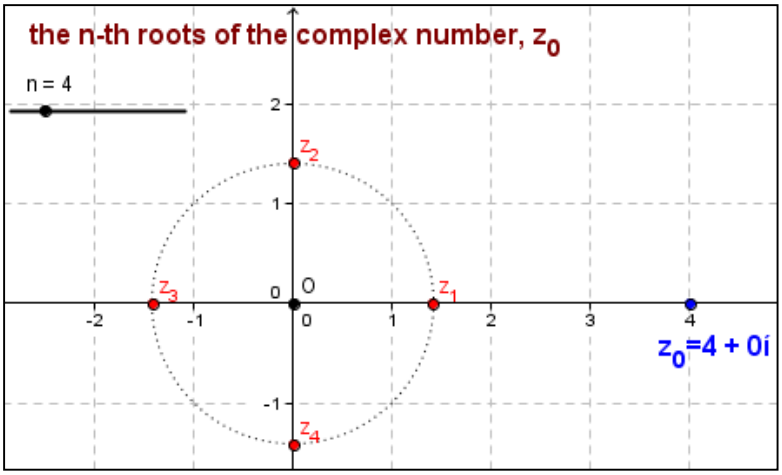

Fig. 2

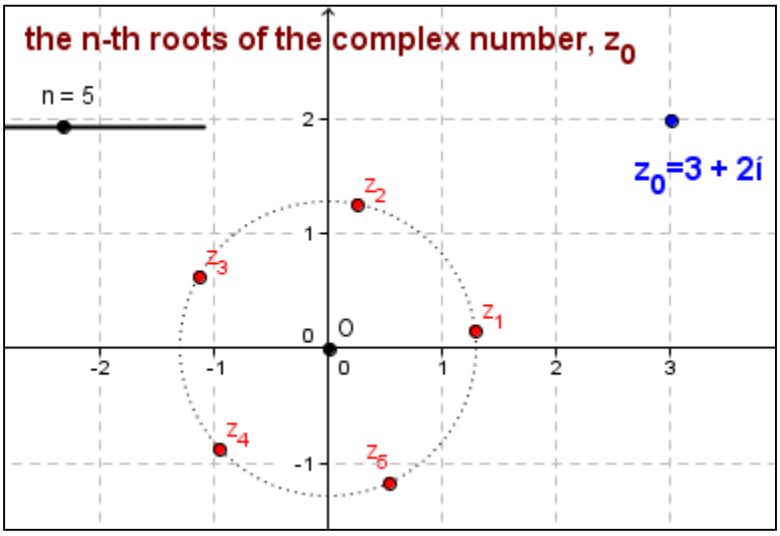

Fig. 3

Note that n is presented with a slider, which gives one a possibility to easily change the value of n and see the new roots corresponding to the new n. One can also change z_0 and follow the changes of the new roots. The fact that the roots are uniformly distributed on the circumference becomes obvious.

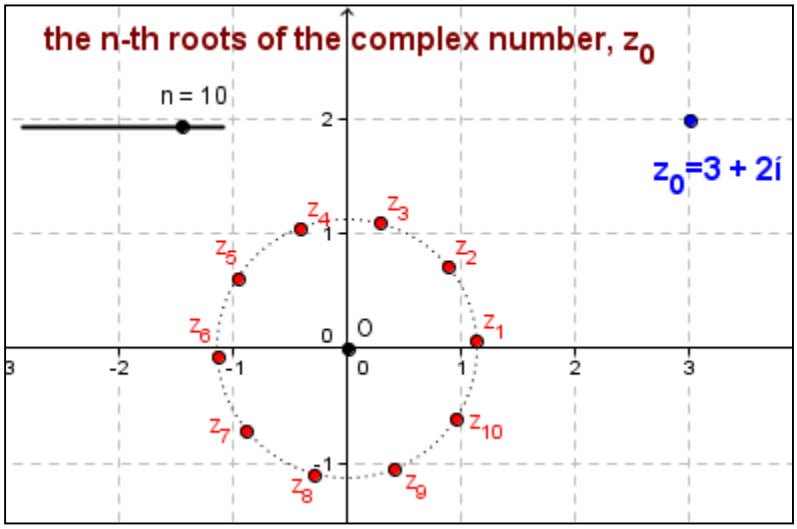

Fig. 4

When it was mentioned about the new interesting and impressive exercises in the textbook, we meant, in particular, the task of creating of such a model and tool. To carry out an exercise like these the student has to be aware of the corresponding commands of the softwares. If this is the case, then the accomplishment of the task will be interesting and attractive and, besides, it will be equivalent of a great group of exercises of finding the roots of complex numbers.

The importance and the direct connection of this theme with the main theorem of algebra are obvious. When teaching this theorem and its consequences the discussion of special examples with their computer illustrations make the perceiving of the theorem much deeper and interesting, stressing its great value and fundamental substantiality.

But the theorem cannot be proved for high school students due to the lack of needed special knowledge; and the illustration of examples of many polynomials with different powers 'with the chalk and blackboard' is practically impossible, especially if one takes into account that there is no general algebraic solution–that is, solution in radicals–to polynomial equations of degree of five or higher (Abel–Ruffini theorem, also known as Abel's impossibility theorem) [5]. For instance, it is impossible to present many examples ('with the chalk and blackboard') of such important consequences as the following statements are:

- An odd power polynomial with real coefficients has one real root, at least.
- If a complex (but not real) number is a root of a polynomial with real coefficients, then its complex conjugate number is a root of the same order.

The situation is entirely different if one uses the possibilities of GeoGebra.

The figures, 5-6, are fragments of an interactive model, which illustrates quadratic polynomials with real coefficients and their roots. The coefficients can be adjusted with sliders, which makes possible illustration of hundreds of examples in a short while.

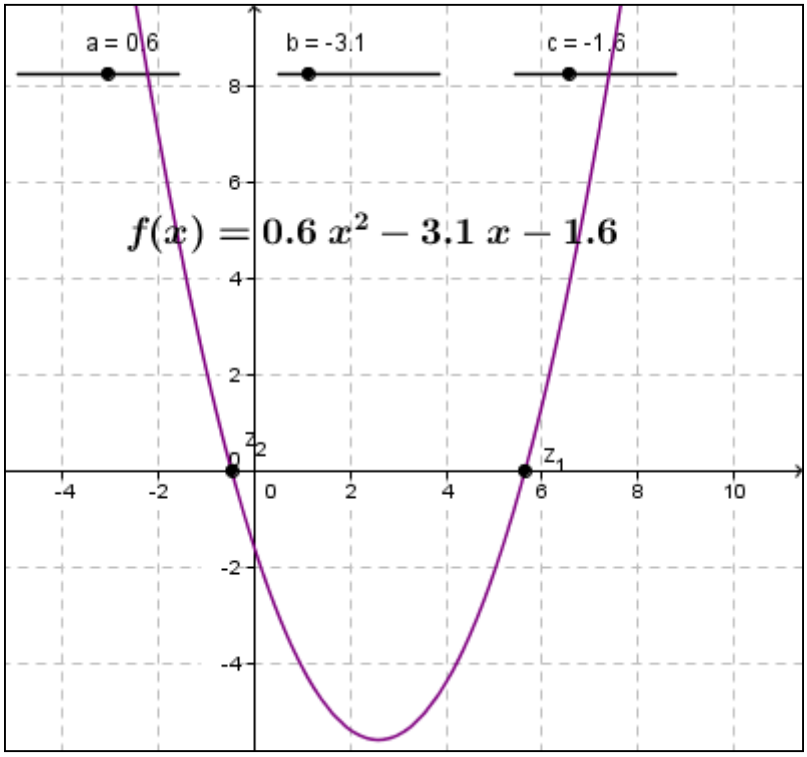

Fig. 5

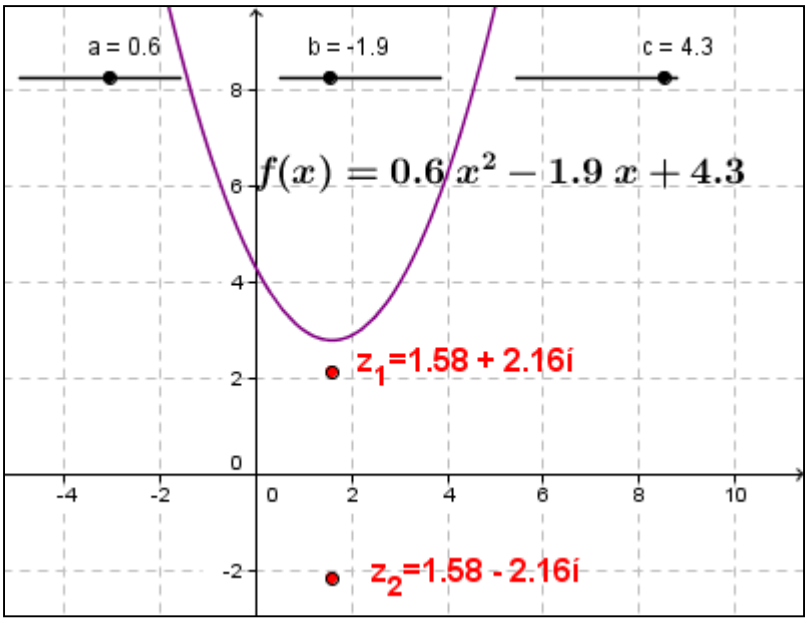

Fig. 6

The figures, 7-8, are fragments of another interactive model, which illustrates cubic polynomials with real coefficients and their roots. Again, the coefficients can be adjusted with sliders.

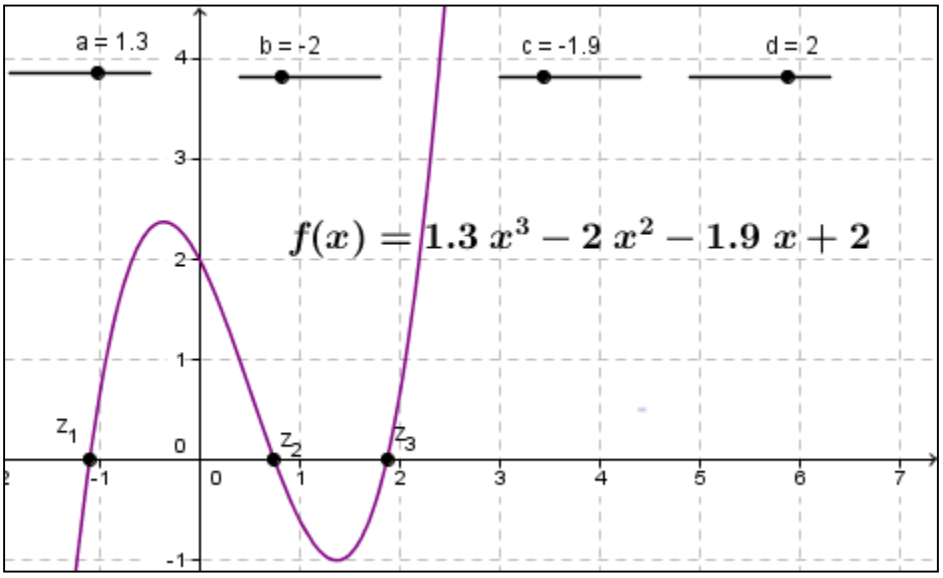

Fig. 7

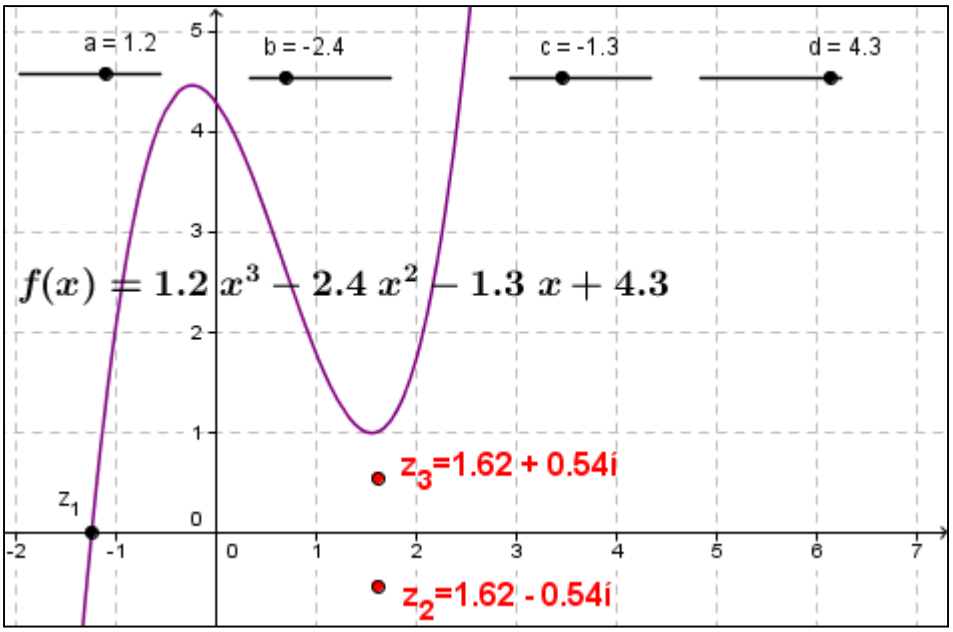

Fig. 8

In figures 6 and 8 the polynomial roots are not only seen as complex plane points, but also their algebraic representations are given, which make obvious the presence of their complex conjugate roots.

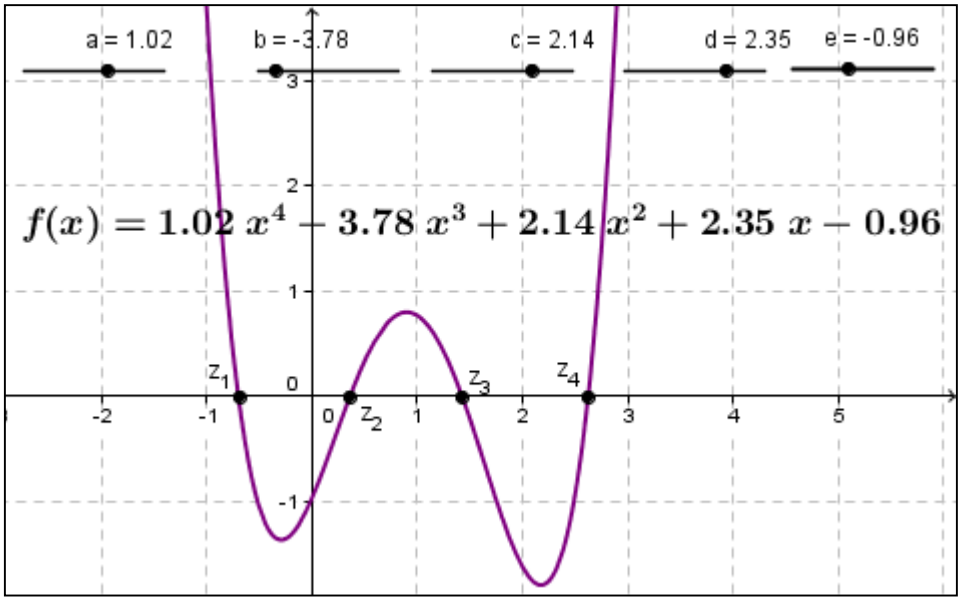

Fig. 9

In figures 9-11 polynomials of fourth power with real coefficients with their roots, correspondingly: having 4 roots, 2 real and 2 complex roots, and 4 complex roots are presented. The corresponding polynomial coefficients, as in the preceding cases, can be adjusted by sliders.

In figures 10-11, it is obvious that every complex root also has its conjugate one, though the complex forms of the roots are not brought.

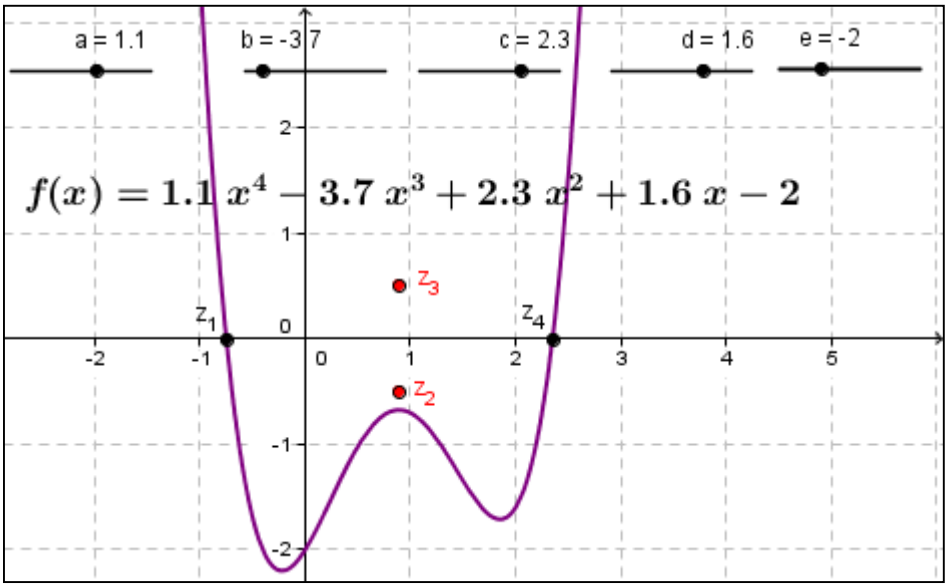

Fig. 10

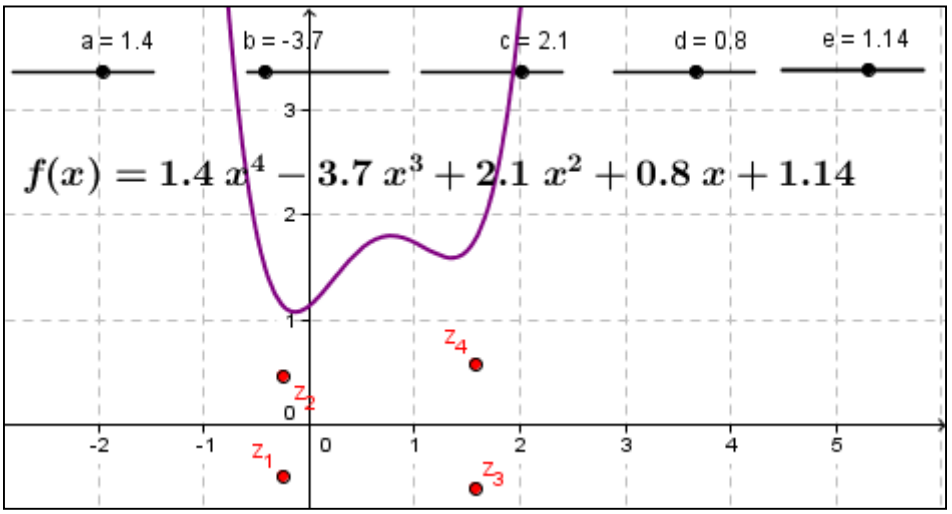

Fig. 11

The next figure, 12, is a fragment of the corresponding interactive model, and it illustrates some polynomials of 7-th power and their roots.

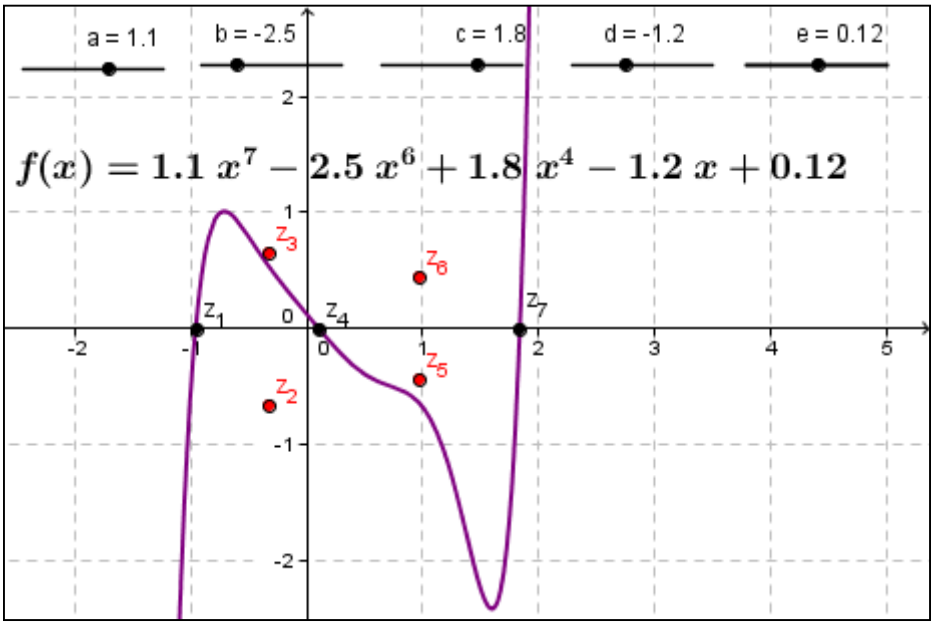

Fig. 12

Creation of interactive models for finding the roots of polynomials with real coefficients, as it was in the case of the interactive model of *n*-th roots of a complex number, is an interesting, attractive and useful task for a student. Such tasks essentially promote their interest to the theme and make their perceiving and learning of the theme deeper, but accomplishment of such tasks is possible only if the students have computers in their classes or in their homes.

It should be added that the students work with computers with more pleasure than with pens and paper. Nowadays, most of students have their home or personal computers and use the internet, and working with them is usual and natural for them, though these methods have been rarely used for teaching intentions in Armenia so far.

Conclusion

The results of the meetings with more than thousand teachers unobjectionably evidenced that the GeoGebra software is a modern, effective and, in many cases, unique tool for teaching and studying mathematics and geometry. It is essential that this software is spread freely, which makes easier its introduction in the education

systems of the countries, which have no financial possibilities of buying analogous non-free softwares.

Teaching the "Complex Numbers" theme through the GeoGebra software makes possible:

- illustration of operations with complex numbers
- illustration of many regularities of complex numbers
- inclusion of the main theorem of algebra and its consequences in this theme and illustration of many and different examples confirming the theorem and its consequences
- offering the tasks of creation of various new models and tools (besides the traditional ones).

All these essentially promote the depth of perceiving and learning of the theme, making the efficiency of teaching much higher.

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