Geogebra as an aid tool for discovering mathematical solutions in teaching and learning of mathematics in Vietnamese schools

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ABSTRACT: In general, deductive reasoning phase emerges in a formal mathematical solution. For this reason, sometimes we do not know why and how mathematical solutions are discovered and created in mathematical literature. In this article, we want to discuss a way of using Geogebra as an aid tool for Vietnamese mathematics teachers to instruct their students to discover and find solutions to mathematical problems.

KEYWORDS: Geogebra, aid tool, mathematical solutions, teaching and learning of mathematics

1. Introduction

In this article, firstly, we describe a typical conventional teaching style in teaching and learning of mathematics in Vietnamese schools. More specifically, when students are not able to find a solution to a given problem, their teachers often try to explain an existent solution to them. After understanding the existent solution to the given problem, the students are asked to solve similar problems so that they can imitate the existent strategy and the existent solution they know or have learnt in similar situations. Generally, the students often find solutions which their teachers explain to them not very natural and not meaningful. Moreover, the students do not usually know and understand why and how mathematical solutions and proofs are discovered, created and found. Secondly, we describe a way in which Vietnamese mathematics teachers can use Geogebra to instruct their students to discover and find a solution to a given problem. With the aid of Geogebra, sometimes a natural solution students find and create is different from existent solutions in mathematical literature.

2. Discovering solutions to mathematical problems with the aid of Geogebra

a. A process of instructing students to discover and find mathematical solutions with the aid of Geogebra

When we read official mathematical literature, we often see “ready-made mathematics” [Fre91]. In mathematical documents, an existent solution to a given mathematical problem does not tell us why and how it is found, created and discovered. Consequently, students often find ready-made mathematical proofs and solutions unnatural and unmeaningful for them. In Vietnam, when students cannot solve a problem, their mathematics teachers tend to explain a ready - made solution to them. Then the students usually imitate the way or strategy in the solution to solve other similar problems. With this instruction style, the students do not have chances to develop their own critical and creative thinking.

Freudenthal stated that school mathematics should be considered as a human activity [Fre91]. He urged that teachers should create situations so that their students can reinvent mathematics [Fre91].

We use a process with the following main steps to help students to discover and find mathematical solutions with the aid of Geogebra:

Step 1. Using Geogebra tools and commands to create mathematical objects and relations

Step 2. Considering specific cases with the aid of Geogebra

Step 3. *Making projections with the aid of Geogebra*

Step 4. *Creating mathematical solutions*

This process is not linear. In other words, sometimes students at one step come back to the previous steps.

b. Examples

In the following section, we will discuss several examples about using Geogebra as the aid tool for finding solutions to some given problems. In each problem, we present a ready-made solution in mathematical literature (books, journals…). Then we analyze the existent solution. After that, we discuss a way to use Geogebra as the aid tool for discovering and finding a solution to the given problem.

Problem 1

Prove that if p and $8p-1$ are prime numbers, and p does not equal 3 then $8p + 1$ is a composite number.

We often find the following ready – made solution to the given problem in Vietnamese mathematical literature:

Solution 1 (An unnatural solution)

Let's consider the product $M=(8p-1)8p(8p+1)$. Among the consecutive positive integers $8p-1, 8p$ and $8p+1$, there exists exactly one number such that it is divisible by 3.

Since p, $8p - 1$ are prime numbers, and the greatest common factor of 8 and 3 is 1, $8p + 1$ is divisible by 3. Moreover, $8p + 1$ is greater than 3 for every prime number p. Therefore, $8p + 1$ is a composite number.

Comments on the solution

The above solution is very interesting. However, this is not a natural solution. First, it is not easy to discover that we should consider the product of $8p-1, 8p$ and $8p+1$. Furthermore, M has several factors such as 2, 3, 4, 6, 8 and 24. Why do we only consider that 3 divides M?

When Vietnamese students cannot solve the above problem, their teachers often try to explain the above ready – made solution to them. Then the teachers ask the students to solve some similar problems which can be solved with similar solutions.

Discovering a solution with the aid of Geogebra

In the following section, we propose a way teachers can use Geogebra to instruct their students to discover another natural and meaningful solution to the given problem.

Step 1: Using Geogebra command to consider p satisfying the problem condition

IsPrime[<Number>] command is used to check whether a positive integer is prime number or not.

Step 2: Considering specific cases of p with the aid of Geogebra

The following Geogebra commands are used:

Opening *Spreadsheet View;*

*NextPrime[<Number>] command (*is used to find the smallest prime number greater than the entered number);

*DivisorsList[<Number>] command (*is used to give the list of all the positive divisors of the entered number, including the number itself).

We consider specific cases of p in which p is greater than 3, and p and $8p - 1$ are prime numbers; the list of the factors of $8p + 1$ (See figure below).

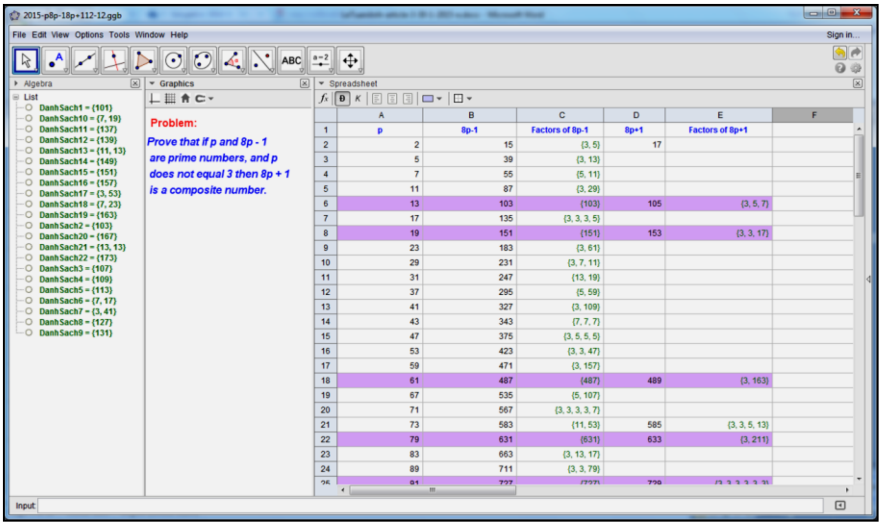

Figure 1: Considering specific cases of p

Step 3: Making a prediction with the aid of Geogebra

The students can discover that 3 always appears in the factor sets of $8p + 1$ when p satisfies the supposition. More exactly, when p satisfies the problem condition, 3 is always a factor of $8p + 1$ (see the column c in the spreadsheet). In other word, 3 always divides $8p + 1$.

Working with Geogebra suggests a prediction: if p is greater than 3, and p , $8p$ - 1 are prime numbers then $8p$ + 1 is divisible by 3.

From this, it suggests that we should consider the remainder when p is divided by 3. So we have the following solution.

Step 4: Creating a mathematical solution (Solution 2)

Since p is a prime number, and p is not equal to 3, if p is divided by 3 then the remainder is either 1 or 2. We consider the following cases:

If the remainder is 1 then $8p$ + 1 is divisible by 3. Moreover, $8p$ + 1 is greater than 3 for every prime number *p*. Thus, $8p$ + 1 is a composite number. In this case, the problem is solved.

If the remainder is 2 then $8p$ - 1 is divisible by 3. In addition, $8p$ - 1 is greater than 3 for every prime number *p*. Consequently, $8p$ - 1 is not a prime number. This contradicts the problem condition.

Problem 2

Find the real value of *m* such that the following equation has only one real root:

$$\sqrt{(4+x)(6-x)} = x^2 - 2x + m \qquad (1)$$

We often find the following solution to the second problem in some Vietnamese mathematical literature:

Solution 1 (An unnatural solution)

If x_0 is a root of the equation (1) then 2 - x_0 is also a root of the equation. If the equation has only one root, we must have x_0 = 2 - x_0. Hence, we obtain x_0= 1. As a consequence, 1 is one root of the equation (1).

By substituting x = 1 in (1), we have $\sqrt{(4+1)(6-1)} = 1 - 2 + m$. Thus, m = 6.

When m = 6, the equation (1) becomes:

$$\sqrt{(4+x)(6-x)} = x^2 - 2x + 6 \qquad (2)$$

We rewrite the right hand side of the equation (2) as $x^2 - 2x + 6 = (x-1)^2 + 5$. We notice that

$$(x-1)^2 + 5 \geq 5 \text{ for every real number } x \quad (3).$$

The equality occurs in (3) when x = 1.

On the other hand, $(4+x)(6-x) = 25-(x-1)^2$. Since $25-(x-1)^2 \leq 25$ for every real number x,

$$\sqrt{(4+x)(6-x)} \leq 5 \text{ for every real number } x \quad (4).$$

The equality occurs in (4) when x = 1.

From (3) and (4), we obtain $\begin{cases} x^2 - 2x + 6 = 5 \\ \sqrt{(4+x)(6-x)} = 5 \end{cases}$. Solve the system of the equations, we have x = 1.

As a consequence, the equation (2) has only one root.

In conclusion, the value of m is 6.

Comment on the above solution

The above solution is also very interesting. However, it is not a natural solution. It is not easy and natural to discover that if x_0 is a root of the equation (1) then 2 - x_0 is also its root.

Discovering a solution with the aid of Geogebra

In the following, we discuss a way mathematics teachers can instruct their students to solve the problem with the aid of Geogebra.

Step 1: Using Geogebra to draw the graphs of the functions

Using Geogebra to draw the graphs of the functions $y=\sqrt{(4+x)(6-x)}$ and $y=x^2-2x+m$:

In *Input Window*, we type $y=sqrt((4+x)(6-x))$ to draw the graph of the function $y=\sqrt{(4+x)(6-x)}$.

Use *Slider tool* to create slider m.

In *Input Window*, we type $y=x^2-2x+m$ to draw the graph of the function $y=x^2-2x+m$.

Step 2: Considering and observing a number of intersections of the two graphs with specific cases of m

While changing the value of m, we observe a number of the intersections of the two graphs.

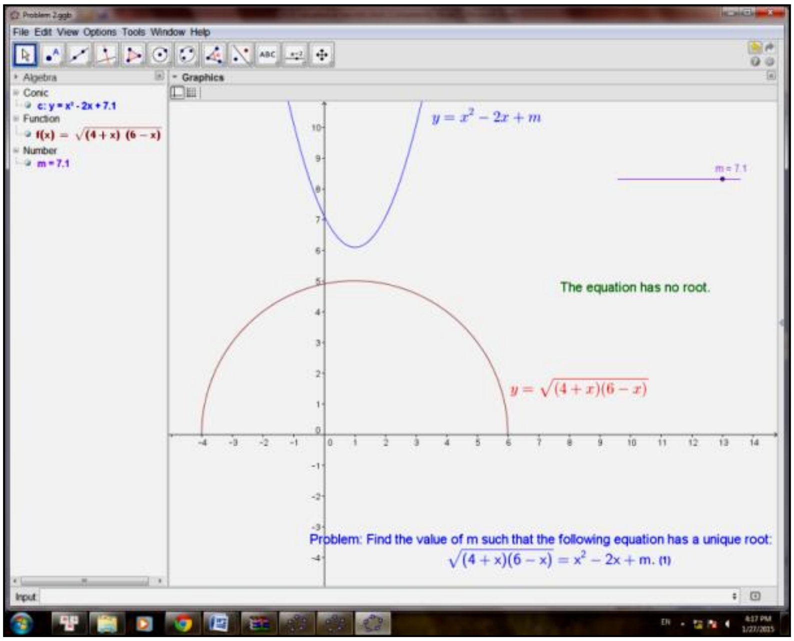

Figure 2: When m is greater than 6, there is no intersection

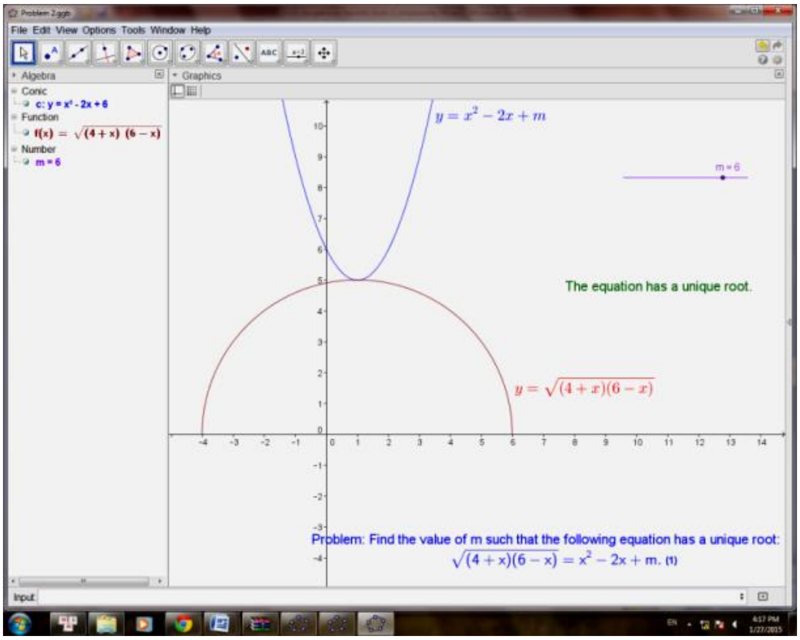

Figure 3: When m is equal to 6, there is only one intersection

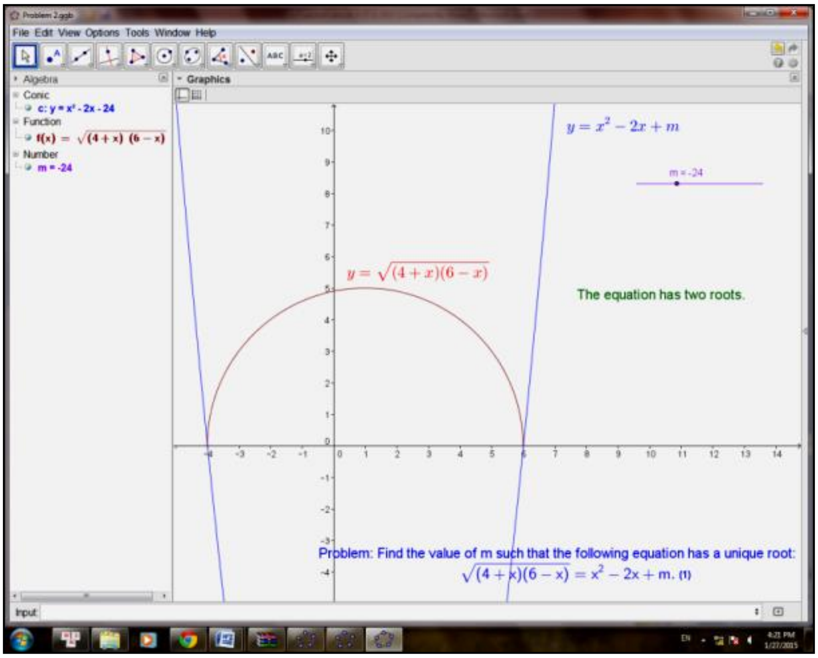

Figure 4: When $m < 6$ and $m \geq -24$, there are two intersections

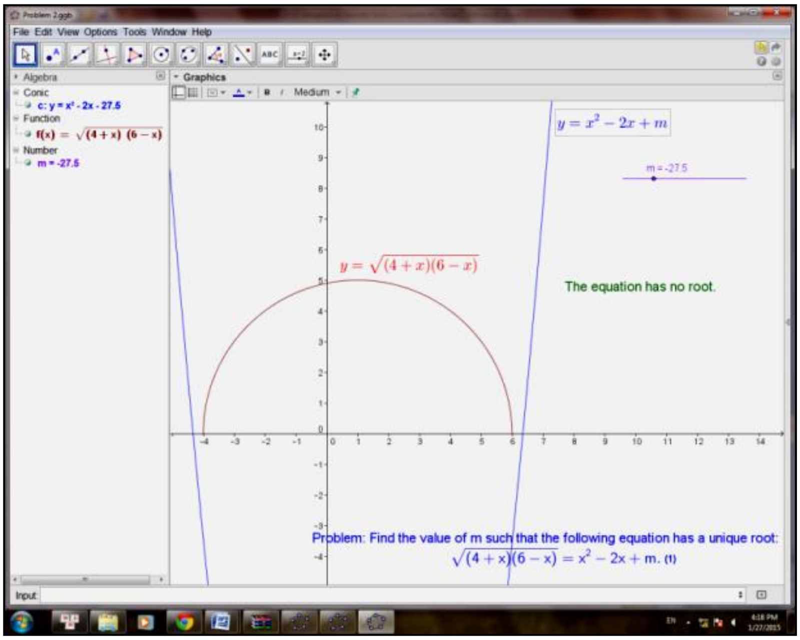

Figure 5: When m is less than -24, there is no intersection

Step 3: Making a prediction with the aid of Geogebra

We observe that the range of the function y = $\sqrt{(4+x)(6-x)}$ is [0, 5]. We also notice that the graph of this function is the semi - circle with the center *I*(1, 0) and the radius 5.

With the above observation, the students can predict that:

- When m is greater than 6, the equation (1) has no root.
- When m is equal to 6, the equation (1) has a unique root.
- When m is less than 6 and greater than or equal to -24, the equation (1) has two roots.
- When m is less than -24, the given equation has no root.

The above observation and prediction suggests the following solution:

Step 4: Creating a mathematical solution (Solution 2)

We denote $\sqrt{(4+x)(6-x)}$ by u. We rewrite the equation (1) as

$$u^2 + u = m + 24, \text{ where } u \in [0;5] \qquad (5)$$

We draw the graph of the function $y = u^2 + u$, where $u \in [0;5]$ and the line $y = m + 24$.

If $m+24 < 0$ or $m+24 > 30$ then the equation (5) has no root in the interval $[0;5]$. Hence, if $m < -24$ or $m > 6$ then the equation (1) has no root.

If $m = -24$ then the equation (5) has root $u = 0$ in the interval $[0;5]$. Thus, the equation (1) has two roots (– 4 and 6).

If $m > -24$ and $m < 6$ then the equation (5) has one root in the interval (0;5). We notice that the graph of the function $y = \sqrt{(4+x)(6-x)}$ is the semi-circle whose center is at (1, 0) and radius 5. Consequently, the equation (1) has two roots.

When m equals 6, the equation (5) has a unique root $(u = 5)$. Therefore, the equation (1) has only one root (x = 1).

In conclusion, the value of m is 6.

With the above solution, we also know that when the equation (1) has either two roots or no root.

Conclusion

With the above examples, we want to show that mathematics teachers can use Geogebra as the aid tool to help their students to discover and find solutions to mathematical problems. Working with Geogebra suggests natural and meaningful solutions to the given problems for students.

References

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