MATHEMATICAL MODELING PROCESS

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Abstract. In teaching mathematical, mathematical modeling is one of the strong tools that promote effective learning. But it must be used properly. Teachers need to use mathematical modeling as a part of their teaching method. Hence, we propose a competence oriented innovation of teaching mathematical through Mathematical modeling skills in the process of solving mathematics and reality problems.

Keywords: *Mathematical modeling, modeling skills, modeling process, mathematics teaching methods.*

1. INTRODUCTION

The word "modeling" comes from the Latin word "modellus". It describes a typical human way of coping with the reality. Although abstract representations of real-world objects have been in use since the stone age, a fact backed up by cavemen paintings, the real breakthrough of modeling came with the cultures of the Ancient Near East and with the Ancient Greek. The first recognizable models were numbers; counting and "writing" numbers (e.g., as marks on bones) is documented since about 30.000 BC (Hermann Schichl, 2000). Many mathematicians took interest in model building. The creation of models such as ours began in the late 19th century, alongside revolutions in algebraic geometry and differential geometry. Many new mathematical objects were discovered in the late 19th century, so mathematicians began building models to demonstrate their properties. At the time, such models depicted objects at the forefront of research on algebraic surfaces. The earliest record of model building dates back to 1873 and deals with a plaster model of Steiner's Roman surface built by German mathematician Ernst Kummer; (Kim and her colleagues, 2010) viewed a mathematical modeling problem as a non-routine problem that involves real-world applications and mathematical concepts that lead to the creation of a mathematical model.

About mathematical models in teaching:

(Maaß, 2006) state that, Modelling is one of a number of reality-based learning activities, it involves simplifying a complex real situation, creating a model, working mathematically with it and interpreting the result in that real situation; Mathematical models are used to interpret real-world situations or non-mathematical situations in mathematical formats (Fox, Watters, 2005). For example, graphs, tables and equations are used to model and make intelligible interpretations of complex relationships among various phenomena; (Ok-Ki Kang, Jihwa Noh, 2012), modeling is a cyclical process of creating and modifying models of empirical situations to understand them better and improve decisions. The role of modeling and teaching mathematical modeling in school mathematics has received increasing attention as generating authentic learning and revealing the ways of thinking that produced it; (Lesh, Lehrer, 2003) assure that the distinction between the model and the world is not merely a matter of identifying the right symbol - referent matches; rather it depends intimately on the accumulation of experience and its symbolic representations over time. We state that, Modelling is one of a number of reality-based learning activities, it to represent and solve real-world problems. Students learn to use a variety of representations of mathematics and to select and apply appropriate mathematical methods and tools in solving real-world problems.

One of the most important aims for construction of models is to define the problem such that only important details becomes visible, while irrelevant features are neglected.

A mathematical model of a complex phenomenon or situation has many of the advantages and limitations of other types of models. Some factors in the situation will be omitted while others are stressed. When constructing a mathematical system, the modeler must keep in mind the type of information he or she wishes to obtain from it.

We build mathematical modeling process for teaching high school in Vietnam.

2. PROCESS OF MODELING

A model is a representation or an abstraction of a system or a process. We build models because they help us to define our problems, organize our thoughts, understand our data, communicate and test that understanding, and make predictions. A model is therefore an intellectual tool. (Cheng, 2001) presented examples of how the process of mathematical modeling could be introduced in the Singaporean secondary classroom using basic mathematical ideas and concepts and found many challenging and exciting skills emerging in developing models which have often been ignored in traditional school mathematics.

Some process of building a model had been proposed:

The process of building a model from a real-world problem is an exciting task. It often involves many iterations in a cycle like in Fig. 1. This is the traditional description of the modeling process, the various stages of the modeling cycle appear interconnected, demanding even more interaction between the subtasks.

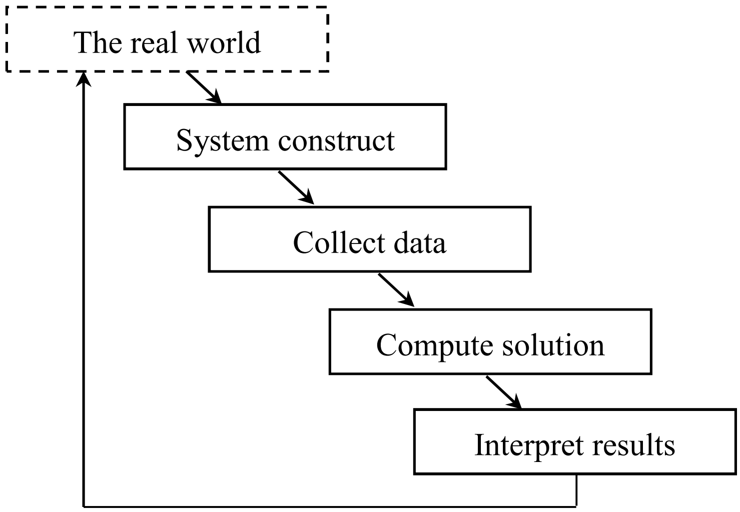

Figure 1. Modeling cycle (by Hermann Schichl)

(Mette Sofie Olufsen, 2003) The teacher begins with some observations about the real world. Teacher wishes to make some conclusions or predictions about the situation he or she has observed. One way to proceed (E) is to conduct some experiments and record the results. The model teacher follows a different path. First, teacher abstracts, or translates, some of the essential features of the real world into a mathematical system. Then by logical argument (L) teacher derives some mathematical conclusions. These conclusions are then interpreted (I) as predictions about the real world. To be useful, the mathematical system should predict conclusions about the real world that are actually observed when appropriate experiments are carried out. If the predictions from the model bear little resemblance to what actually occurs in the real world, then the model is not a good one. The modeler has not isolated the critical features of the situation being studied or the axioms misrepresent the relations among these features. On the other hand, if there is good agreement between what is observed and what the model predicts, then there is some reason to believe that the mathematical system does indeed capture correctly important aspects of the real-world situation (Fig. 2).

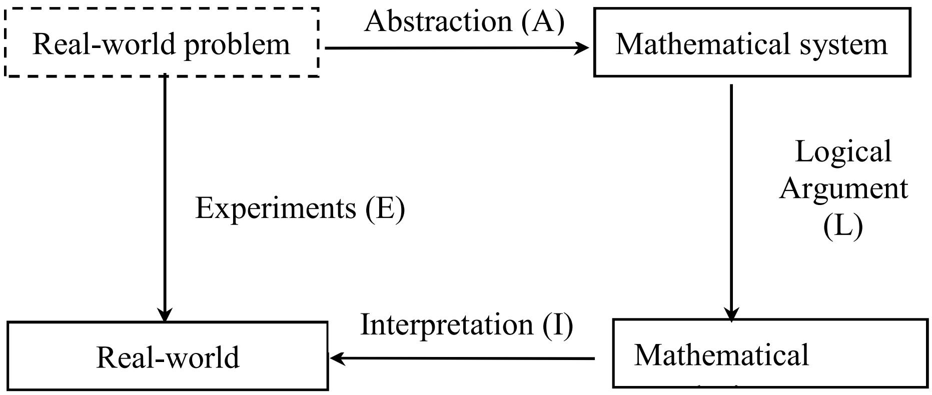

Figure 2. Modeling cycle (by Mette Sofie Olufsen)

From a review of the Modeling (Ok-Ki Kang, Jihwa Noh, 2012; Mette Sofie Olufsen, 2003). We propose a mathematical model that for teaching in Vietnam (Fig. 3).

(1) Abstraction in its main sense is a mathematical conceptual process by which general rules and concepts are derived from the usage and classification of specific features. We identifying variables in the situation and selecting those that represent essential features.

(2) Applying mathematics knowledge: teacher build effective links between the mathematical equations used to solve problems in the real world of objects. Through the analysis of mathematical related frameworks. Analyzing and performing operations on these relationships to draw conclusions; if the implementation of the performed operations cannot be complete, then revise the selection of the variables used to formulate the model. Therefore, we have mathematical conclusions.

(3) Mette Sofie Olufsen (2003) state that, these conclusions are then interpreted as predictions about the real world. To be useful, the mathematical system should predict conclusions about the real world that are actually observed when appropriate experiments are carried out (1') and interpreting the results of the mathematics in terms of the original situation.

(4) Expansion Real-world problem: Tackling real world problems can make sustainability issues more tangible and meaningful to students. Real examples provide concrete applications to mathematical knowledge and skills learned in the classroom as they relate to students themselves and society. Real examples also encourage students to be aware of the choices they make and how they fit into a greater societal context. Real world examples demonstrate the complexity and unpredictability of real issues,

and as such, can stimulate critical thinking. They also highlight the need for an inter- and multi-disciplinary approach to problem solving. Further, using examples from the real world demonstrates that, oftentimes, there is no perfect solution to a given problem. But, in doing so, gets students thinking about solutions, rather than just focusing on problems.

3. TEACHING MODELING MATHEMATICS IN HIGH SCHOOL IN VIETNAM

When assessment the modeling problems and activities used in those studies were tasks that were carefully developed for research. Those Real-world problems were not presented in their mathematics textbooks. This creates a complex situation in

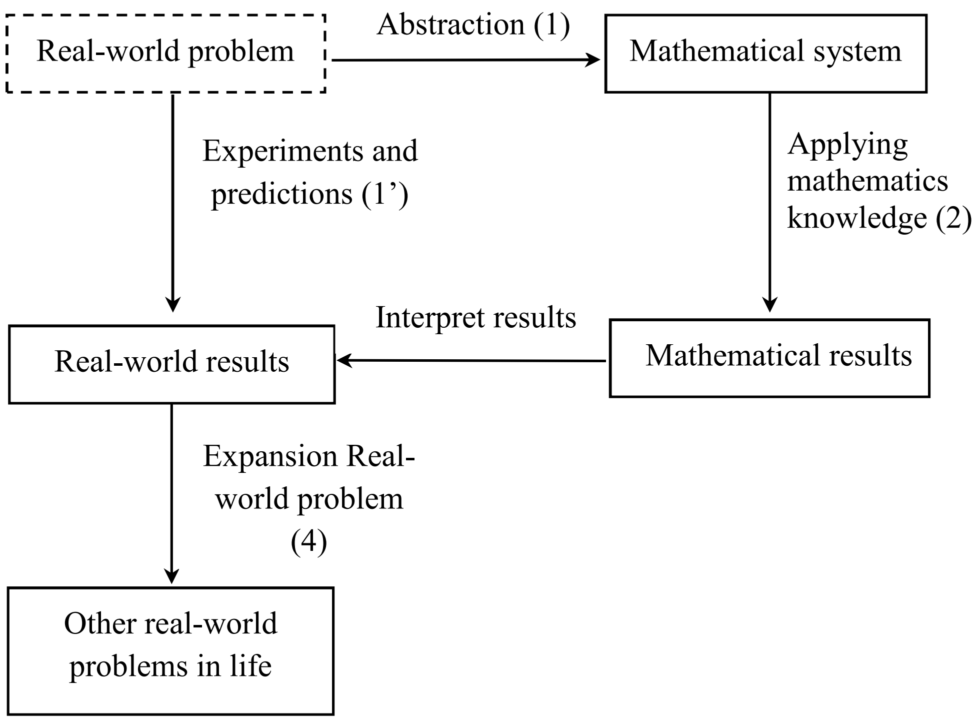

Figure 3. Modeling cycle (by L.H. Quang)

which classroom teachers need to search for or develop such activities themselves if they want to use them in their instruction. Although a teacher's role as a researcher should not be overlooked, such a demerit discourages or creates resistance towards using modeling problems in their classrooms. To address this, we present our attempts to transform traditional mathematical teaching in Vietnam into more authentic modeling problems

Many current reform recommendations value the mathematical modeling of phenomena as one of the most powerful uses of mathematics, and emphasize modeling and contextualized problem solving across the mathematics curriculum in Vietnam. The teacher makes choices of which problems to engage students in, a teacher's capacity to use and appreciate the importance of the mathematical models in varying contexts is critical.

3.1. Levels of modeling problems

From a review of the literature (Galbraith, Clatworthy, 1990; Ok-Ki Kang, Jihwa Noh, 2012). We based on the completeness and ambiguity of the information composing a problem, modeling problems can be categorized into three levels, as follows:

• Level 1*:* Problems at this level are already carefully defined so there is little ambiguity about what needs to be done and how to do it. They contain all the information necessary to formulate a model (Ok-Ki Kang, Jihwa Noh, 2012). Students are expected to search for the needed mathematics knowledge that is hidden in the problem, recall the (implicitly or explicitly) called for procedure, and carry it out correctly. There is no need to collect additional data to formulate a model.

• Level 2: Problems at this level still have a little ambiguity about what needs to be done and how to do it. However, Real-world problem do not provide all the information needed to successfully complete the task. Although students may be given a direction of what data is needed, they need to devise a meaningful way to gather the needed data and test if the gathered data would produce a reasonable answer.

• Level 3: Problems at this level are comprised of information that is open-ended, incomplete and/or redundant. There is not a well-rehearsed approach or pathway explicitly suggested by the task. Students are expected to analyze the task to find what needs to be done and actively examine tasks constraints that may limit or suggest possible solution strategies and solutions.

3.2. Some practical examples of mathematical modeling

In this section, Some of these examples are adapted from sources such as (Swetz, Hartzler 1991; Wood, 1992; Blane, Evans, 1993; Ang Keng Cheng, 2001; Tran

Trung Tinh, 2004; Le Hong Quang, 2015, et al.,) and using modeling cycle by L.H. Quang (Fig 3.).

Example 1. If one wants to measure the distance d from shore to a remote ship via triangulation, one marks on the shore two points with known distance l between them (the baseline). Let α, β be the angles between the baseline and the direction to the ship (Fig. 4).

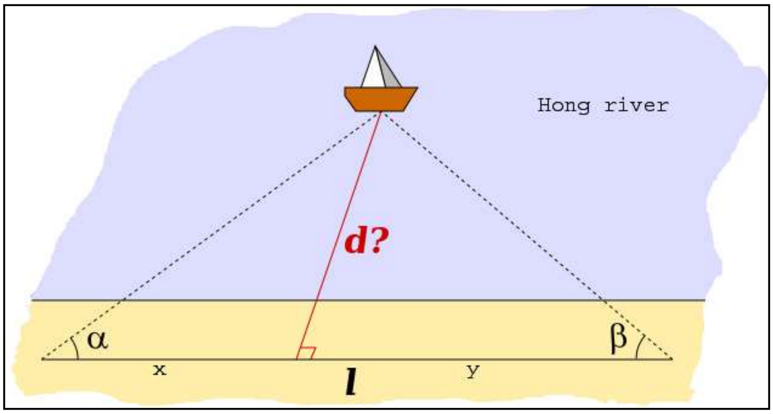

Figure 4. Ship models on the river

Real-world problem: we want to measure the distance from shore to a remote ship.
Mathematical system: Let triangle has a side and the two angles adjacent to it.
Applying mathematics knowledge:
The standard method of solving the problem is to use fundamental relations.
From the formulae above (ASA case) one can define the length of the triangle height:

$\tan\alpha = \frac{d}{x}$, $\tan\beta = \frac{d}{y}$, where $0 < x, y < l$ and $x + y = l$ we deduce results.

Mathematical results: $d = \frac{\sin\alpha\sin\beta}{\sin(\alpha+\beta)} l = \frac{\tan\alpha\tan\beta}{\tan\alpha + \tan\beta} l$

This method is used in cabotage. The angles α, β are defined by observation of familiar landmarks from the ship.

Other real-world problems in life: As another example, if one wants to measure the height h of a mountain or a high building (Fig. 5), the angles α, β from two ground points to the top are specified. Let l be the distance between these points. From the same ASA case formulas, we obtain:

$$h = \frac{\sin\alpha \sin\beta}{\sin(\beta-\alpha)} l = \frac{\tan\alpha \tan\beta}{\tan\beta - \tan\alpha} l.$$

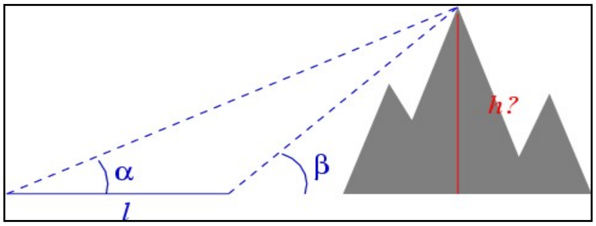

Figure 5. Measure the height of a mountain

Example 2: *Design intent a spot for bus stop*

Design intent: This design of providing seating that serves bus riders. This stop is at one of the major intersections in the Central Soc Son neighborhood. It is close to Xuan Giang high school, town library (Fig. 6). Find a spot for bus stop so that the total distance between AC and BC is smallest

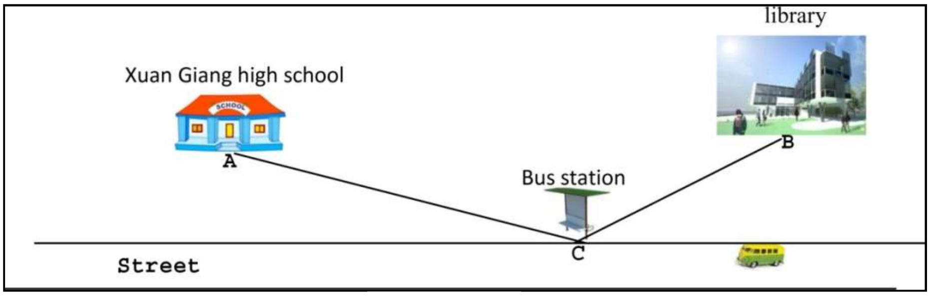

Figure 6.

Real-world problem: We wants to find a spot for bus stop so that the total distance from school to bus station and from library to bus station is smallest.

Mathematical system: Transform reflect, how to find the sum of line segments, the sum of line segments is smallest.

Applying mathematics knowledge:

The standard method of solving the problem is to using transform reflect and the sum of the length of line segments A'C and BC.

$$AC + BC = A'C + BC \geq A'B$$

We deduce $\min\{AC + BC\} = \min\{A'C + BC\} = A'B$. Let $C' = A'B \cap d$, where d is road pavement (Fig. 7.) *Mathematical results:* the stop bus spot is C' .

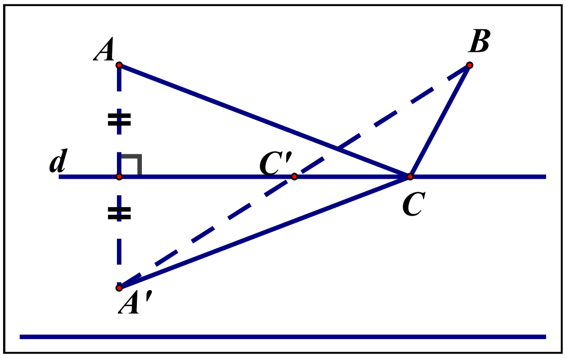

Figure 7.

Other real-world problems in life: Design intent: This design of providing bridge that serves people. This bridge is at one of the major intersections from Longbien district to opera house the in Hanoi center (Fig. 8). Find a spot to build the bridge so that the total distance from Longbien district to the opera house is smallest (including segments lines: $AB + BC + CD$).

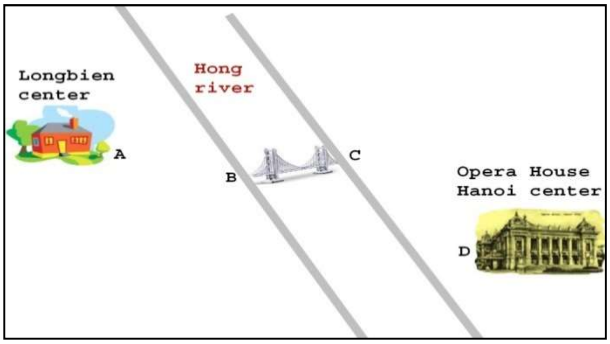

Figure 8. Building the bridge

Example 3: Biggest box problem

Suppose we intend to make an open-top box using a square piece of card of side s by cutting a square (of side, say x) from each corner of the card. The resulting piece is then folded to form the box.

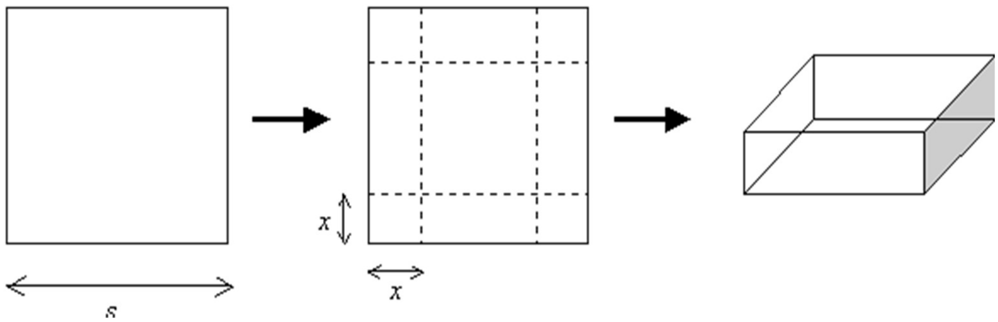

Figure 9. The empirical model of the boxes

The question is: what should x be if we wish to make the biggest box (in terms of volume)?

There are several approaches to this problem. Here, two are described.

Method 1. Empirical approach

The empirical model involves actually constructing the boxes and taking measurements. This has to be done systematically just like in performing a scientific experiment. Since we are particularly interested in the relationship between the size of the smaller square (i.e. x) and the volume of the box, we systematically make boxes using different values of x.

The sides of the box can then be measured and volume calculated for each case. Alternatively, the volume may be estimated by first pouring sand to completely fill the box. The amount of sand used can be measured using a measuring cylinder. Still another variant could be to weigh the sand instead. Whichever approach is used; the results can be presented in the form of a graph (Fig. 10):

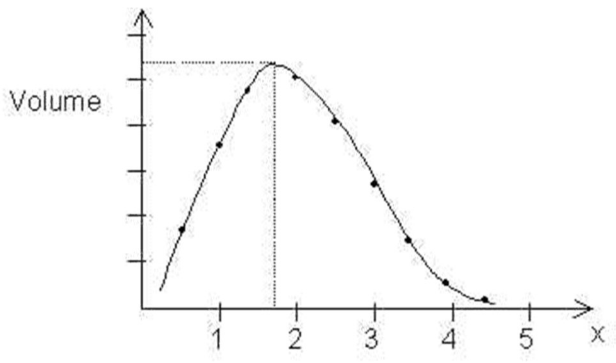

Figure 10: Graph of Volume against x (from data)

A "curve of best fit" is then sketched to locate and estimate x that gives the maximum volume.

Method 2. Analytical approach

An analytical or theoretical model may also be constructed to solve the problem. This approach is more abstract and involves the use of algebra and geometry. We model the box by a geometric diagram (such as the one in Fig. 10). We then find the volume of the box in terms of the dimensions' s and x. It is not hard to see that the volume of the box, V is given by $V = x(s-2x)^2$ or $V = 4x^3 - 4sx^2 + s^2x$

Suppose the original square cards have sides of dimension, say, s = 10 cm. Then, we have $V = 4x^3 - 40x^2 + 100x$.

This is perhaps a good point to introduce the cubic function. In this particular case, the function models the relationship between the volume of the box and the size of the cut-off square. It now remains for us to find the value of x that makes V maximum. How this is done depends on the mathematical ability and maturity of the learner. For instance, a student familiar with calculus may choose to find the derivative and the turning point of the function to obtain the maximum. Another may use a graphing tool to plot a graph of V against x to estimate the maximum. Fig. 11 shows a plot generated from the popular graphing tool.

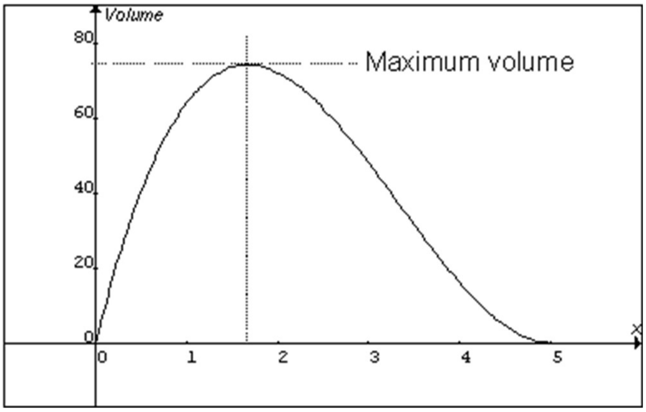

Figure 11: Graph of $V = 4x^3 - 40x^2 + 100x$ (from graphing tool)

4. CONCLUSION

Research has shown process of modeling that we use to help students use their knowledge of mathematics to solve real world problems. Teachers can create examples with different levels that will improve students' confidence when tackling 'open' tasks. Research results confirm the role and the effectiveness of modeling activities in developing students' mathematical literacy in Vietnam.

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